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ABSTRACT

How seven pivotal skills in elementary school mathematics are treated in five current (1981-1983) textbook series for grades 2-4 is described. The report considers the amount of instruction and practice provided, where it is located in the textbook's year-long sequence of lessons, and the extent to which practice is concentrated in one place or distributed across several units of instruction. Seven tables provide a summarized overview, followed by a discussion, for each of the following areas: addition and subtraction with regrouping, word problems involving addition and subtraction, multiplication and division hard facts, word problems involving multiplication and division, fraction recognition, multiplication and division with one-digit multipliers or divisors, and equivalent fractions. Appendix A contains full data on each textbook series at each appropriate grade level, while Appendix B contains lesson time lines which show exactly where the pivotal instruction occurs. (MNS)

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PIVOTAL SKILLS IN ELEMENTARY MATHEMATICS INSTRUCTION

Jannine Perkins

ABSTRACT

This report describes how seven pivotal skills in elementary school mathematics are treated in current textbooks. The report considers the amount of instruction and practice provided, where it is located in the textbook's year long sequence of lessons, and the extent to which practice is concentrated in one place or distributed across several units of instruction.

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PIVOTAL SKILLS IN ELEMENTARY MATHEMATICS INSTRUCTION

Jannine Perkins

Whenever elementary school students require serious remediation, the problem has inherent antecedents in earlier grade levels. What was planned as regular instruction can't go on for these students, because some skills that were assumed to have been taught at earlier grade levels are not in place. Somewhere, at an earlier grade level, instruction on some pivotal skill was not provided, or was provided but was not concentrated enough, and now, at a later grade level, instruction that depends on that pivotal skill can't go on.

The most desirable way to handle remediation is to avoid the need for it by reducing the risk that instruction on pivotal skills in the early grades is too limited or, worse yet, missed entirely. Instructional risk reduction, as an alternative to remediation, is a new concept (Buchanan, Schutz, and Milazzo). What distinguishes a skill as pivotal in grade-by-grade instruction involves more than the amount of linkage that exists between that skill and other important skills that are developed at subsequent grade levels. Many skills at one grade level are linked closely to development of other skills at other grade levels, but some represent greater risks to successful grade-by-grade instruction than others because of the way they are presented in most textbooks in the cycle of introduction, review, and extension across two or more grade levels.

This paper will discuss how seven elementary mathematics pivotal skills are treated in mathematics textbooks. "How" pivotal skills are treated can most unambiguously be described along two dimensions:

1. how much instruction is found in the text; and
2. where the instruction is located.

The how much dimension is important because it describes the amount of potential focus time available for students. Focus time is lesson time devoted to concentrated instruction and practice on a specific skill. This focus time is only potential in the sense that the instruction is in the textbook and "available" for teachers and students to use. SWRL inquiry confirmed by Freeman, et al. (1983) shows that teachers typically do a fair amount of picking and choosing of topics to include or not to include during the school year. Quite frequently the content not included is that found in the final two or three chapters (anywhere from 1/4 to 1/6) of the textbook. This means that the "where dimension" is also important. If instruction on a pivotal skill is found only in the final fourth or sixth of the textbook, it will not be assigned to some students at all during that school year.

The present study inquired how much and where instruction on pivotal skills is found in five of the "newer generation" of elementary mathematics textbooks:

Scott Foresman Mathematics
Scott Foresman Company, 1983

Mathematics In Our World
Addison-Wesley Publishing Company, 1981

Heath Mathematics
D.C. Heath and Company, 1981

Mathematics for Mastery
Silver Burdett Company, 1981

Macmillan Mathematics
Macmillan Publishing Company, Inc., 1982

These textbooks are among the most widely used by schools, and they are among the state adoptions in SWRL's region. The pivotal skills examined and the applicable texts are:

Grade 2	Addition and Subtraction Algorithms with Regrouping (Grade 2 and 3 Texts)
Grade 2	Word Problems Involving Addition/Subtraction (Grade 2 and 3 Texts)
Grade 3	Multiplication and Division Hard Basic Facts (Grade 3 and 4 Texts
Grade 3	Word Problems Involving Multiplication/Division (Grade 3 and 4 Texts)
Grade 3	Fraction Recognition (Grade 2, 3 and 4 Texts)
Grade 4	Multiplication and Division Algorithms with 1-Digit Multipliers/Divisors (Grade 3, 4 and 5 Texts)
Grade 4	Equivalent Fractions (Grade 3, 4 and 5 Texts)

The texts were analyzed for the following:

1. **The <u>average number of lessons</u> focused on a pivotal skill (across textbook series) at the critical grade level and the succeeding grade level**

   Each text at each grade level was divided into "facing" textbook pages, the typical unit for a daily mathematics lesson. The number of whole (two page) or half (one page) lessons focused on the pivotal skill was counted for each series. Then the average number of lessons was computed and listed in the table. Some texts contain occasional cumulative review pages in the text, mostly used for a "change of pace." These are not included in the following count of <u>instruction</u> lessons.

2. **The range in number of lessons focused on a pivotal skill for the five textbooks analyzed**

3. **The average percent (%) of instruction focused on a pivotal skill (across textbook series) at the critical grade level and the succeeding grade level**

   The number of lessons in the five texts analyzed varied from 140 to 190 lessons at a grade level. The average percent of instruction on pivotal skills in relation to the total number of lessons in the text was determined.

4. **The range in percent of instruction focused on a pivotal skill for the five textbooks analyzed**

5. **The most common location for instruction focused on a pivotal skill (across textbook series) at the critical grade level and the succeeding grade level**

   The textbook at each grade level was divided into sixths. The most common location in the textbook for the instruction on a pivotal skill was determined (e.g. first means first sixth of the textbook).

The seven tables that follow provides a summarized overview of this information for each of the pivotal skills. Appendix A contains tables which provide the same type of information for each textbook series at each appropriate grade level. Appendix B contains lesson time lines which show, "visually," at each grade level, exactly where the pivotal skill instruction occurs.

Table 1

Grade 2: Addition and Subtraction with Regrouping

Grade Level	Average Number of Lessons	Range	Average % of Instruction	Range	Most Common Location in Text
2	16.6	10-26	10.6%	7-16%	4-5 (2 digit numbers) 6 (3 digit numbers)
3	16.5	7-26	9.6%	4-15%	2-3

At grade 2 all textbooks provide instruction on addition/subtraction with 2-digit numbers and regrouping. Three of the texts include addition/subtraction with 3-digit numbers and regrouping, but this instruction occurs in the final sixth of the book. In three of the series the instruction on the skill is fairly well standardized, appearing in approximately the same location in the textbook. One series presents this instruction early, in the 3rd and 4th sixths of the book, and one series delays serious treatment of the skill until grade 3, with the only instruction found at the very end of the book (see Appendices A and B for detailed listings.) Three of the series have almost exactly the same percent of instruction devoted to this skill at grade 2 (9%, 10% and 11%). The series that delays treatment until grade 3 has 7% of instruction devoted to addition/subtraction with regrouping (and this is found in the final lessons in the book). One series has an expanded focus on this skill, with 16% of grade 2 instruction allotted to the skill; this series is one of the few that contains a section on mixed addition/subtraction exercises.

At grade 3, the location of instruction on this skill has "moved forward" in the textbook, so far forward that two of the series complete instruction on this skill during the 2nd sixth of the text. At this grade level all textbooks include 2- and 3-digit numbers in the algorithms; three of the books extend the skill to use of 4-digit numbers. In grade 3, the instruction on addition/subtraction algorithms with regrouping usually follows directly after a review of addition/subtraction basic facts and the addition/subtraction algorithms without

regrouping. Instruction usually consists of a quick review and then some practice pages. Again the instruction appears fairly well standardized across the series, with two exceptions. Three series devote about the same percent of their instruction to the skill (8%, 10%, 11%). The series that delayed instruction right up to the end of grade 2 devotes 15% of instruction at grade 3 to this topic. At the other extreme, one series devotes just 4% of instruction to the skill. While most series have completed focus on this skill by midyear (the 2nd or 3rd sixth of the text), one series delays work on 3-digit addition and subtraction until the 4th sixth of the text, although 2-digit numbers are taught in the 2nd sixth of the text.

Table 2

Grade 2: Word Problems Involving Addition/Subtraction

Grade Level	Average Number of Lessons	Range	Average % of Instruction	Range	Most Common Location in Text
2	8.6	6-10.5	5.8%	4-7%	1-6
3*	15.1	9-21	9%	5-12%	1-6

*The Grade 3 statistics are inflated in one sense since they represent the number of lessons in which addition/subtraction word problems are included. Approximately 59% of these lessons focus on addition and/or subtraction. The remaining 41% focus on multiplication and/or division or problem solving in general.

The most conspicuous aspect of the instruction on word problems involving addition/subtraction is its fragmentation. In grade 2, lessons are found scattered throughout the textbook. With the exception of two series, lessons on word problems in grade 2 are about equally divided among the following three types of lessons:

1. One page of word problems, usually facing a page with "straight" computation items.
2. Two pages (one entire lesson) of word problems, usually following a set of lessons that have focused on the computation needed to solve the problems.
3. Four or more pages (two to four lessons) of word problems, usually found in a chapter or section focusing entirely on applications or problem solving; these chapters or sections usually follow whole chapters that have focused on computation.

One series does devote instruction almost exclusively to one-page offerings of word problems. Another series devotes instruction exclusively to two-page sets of word problems.

At grade 2, most textbooks feature rebus problems. In these problems key words are replaced by illustrations. Only two textbooks include "word" problems as such at this grade level.

All grade 2 texts include word problems that deal with addition using expressions like "How many does he/she have in all?" Subtraction is mostly "take away" ("I see 3. 2 fly away. How many are left?"). Four textbooks also work on comparison subtraction ("I have 3. You have 2. How many more do I have?"), but only two textbooks have a whole lesson devoted to developing the concept of comparison subtraction.

At grade 3 the instruction on word problems begins to change. The instruction is still fragmented throughout most textbooks. But, at grade 3, word problem instruction goes from a focus on addition/subtraction to a new focus on multiplication or division. In most of the textbooks, the focus on addition and subtraction as the operations used in problem solving begins to dwindle about mid year (the end of the 3rd, or midway through the 4th, sixth of the text). At that point the focus begins to shift to multiplication and then to division basic facts as the operations needed to solve the problems. Addition and subtraction problems are still found, but mostly as contrasting problems for lessons that focus on multiplication and division.

Another change that begins to occur with word problem instruction in grade 3 is the <u>way</u> textbooks handle problem solving lessons. Two textbooks introduce theme lessons (conservation, careers, ecology, etc.) at the end of each chapter or unit; these are in add tion to regular lessons on problem solving that follow computation. One textbook has entire chapters that involve various types of applications, while two follow the more conventional method of placing problem-solving lessons directly after lessons focusing on the required computation.

In grade 3, different textbooks vary in the amount of instruction focused on word problems involving addition/subtraction. Three textbooks devote approximately 10% to 12% of their time, while two textbooks use about 5% to 6% of their time. In grade 3 all textbooks feature word problems with "take away" and "comparison" subtraction.

Several textbooks occasionally provide two or three word problems at the end of a computation lesson. These lessons are not counted here as

problem-solving lessons, because the main objective of the lesson is to teach computation--work on word problems is incidental.

Table 3

Grade 3: Multiplication and Division Hard Facts

Grade Level	Average Number of Lessons	Range	Average % of Instruction	Range	Most Common Location in Text
3	9.6	5-13	5.6%	3-7%	3 or 4 (x) 5 (÷)
4	4.7	3.5-6	2.6%	2-3%	2 (x) 2 (÷)

Instruction on "hard" multiplication and division facts involves lessons that teach multiplication with the numbers 6, 7, 8, and 9 by the numbers 6, 7, 8, and 9 and their related division facts (36 ÷ 6, 48 ÷ 8, and so on).

The data in Table 3 should be interpreted in light of the typical sequence in elementary mathematics textbooks which is somewhat like the following:

Lesson — Multiplication by 6
Lesson includes 1, 2, 3, 4, 5, 6, 7, 8, or 9 x 6

.
.
.

Lesson — Multiplication by 9
Lesson includes 1, 2, 3, 4, 5, 6, 7, 8, or 9 x 9

.
.
.

Lesson — Division by 6
Lesson includes 6, 12, 18, 24, 30, 36, 42, 48, 54 ÷ 6

.
.
.

Lesson — Division by 9
Lesson includes 9, 18, 27, 36, 45, 54, 63, 72, 81 ÷ 9

With this sequence, two things occur. First, each lesson on "hard" facts includes both hard facts and a lot of easier ones. In fact, in each lesson listed in Table 3, approximately 1/2 to 2/3 of the problems involve the easier facts. Second, the multiplication facts and the division facts are not typically covered in the same set of lessons. In fact, with the exception of one textbook at each grade level (although it is a different textbook in grade 3 than in grade 4), many lessons come between the instruction on hard multiplication and hard division facts (10, 14, 40+ and 60+ lessons). Thus, while the relationship between easy division facts and easy multiplication facts (as related operations) is emphasized in the text, the instruction on hard facts relies more on "memorization."

As can be seen from Table 3, the location of the instruction moves forward in the text from grade 3 to grade 4. By grade 4, instruction is about half of what it was in grade 3 and consists mainly of a quick review of the facts just before instruction begins on the multiplication and division of larger numbers by 1-digit multipliers or divisors.

Table 4

Grade 3: Word Problems Involving Multiplication/Division

Grade Level	Average Number of Lessons	Range	Average % of Instruction	Range	Most Common Location in Text
3*	9	6-11.5	5.4%	4-7	3 or 4-6
4*	9.1	5.5-13.5	5.2%	3-8%	2 or 3-6

The instruction on word problems involving multiplication and division is usually presented in one of three formats: one-page half lessons: a single two-page lesson; or a set of two or three lessons (four to six pages). In grade 3, the predominant format is the single, two-page lesson found either directly after a lot of instruction on straight computation, in an entire chapter devoted to problem solving or in a theme lesson ("conservation," "careers," etc.). Again, as with addition/subtraction word problems, the instruction tends to be fragmented. Unlike the early work on addition or subtraction word problems, there are few "rebus" problems that use multiplication or division in grades 3 and 4.

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*As with word problems involving addition and subtraction, some numbers in this table are inflated. In grade 3, for example, about 60% of the lessons include addition, subtraction, multiplication and division problems, although the focus is usually more on multiplication and division than on addition or subtraction.

The data on multiplication and division word problems enumerates them according to three types that are fairly standard in basic textbook instruction:

> multiplication (9 teams; 5 children on each team; how many children in all)
>
> partitive division (45 children; 5 teams; how many on each team)
>
> measurement division (45 children; 9 on each team; how many teams)

The data below list the average number of problems, and the range, for each type found in instruction, across grade levels.

	Multiplication Averages		Division Averages			
			Partitive		Measurement	
Grade Level	Average Number of Problems	Range	Number	Range	Number	Range
3	34.6	22-40	10.6	8-14	19.6	4-35
4	44.4	24-99	11	2-24	10.2	2-28

The numbers in this table represent the average and range of the approximate number of word problems of each type found in half (one page) or whole (two pages) lessons in the texts. Across all textbooks and both grade levels, there are two instances in which an equal number of multiplication and division problems are included. In one textbook, more division than multiplication problems are included. In all other instances there are more multiplication problems than there are division

problems of either or both types in the instruction. In grade 3 approximately 53% of the problems involve multiplication, 16% of the problems involve partitive division and 30% of the problems involve measurement division. In grade 4, the proportion of multiplication problems has increased to 66%, partitive division stays at 16% and measurement division decreases to 18%.

The count given above does not include the occasional two or three word problems found at the end of some computation lessons. A quick analysis was performed to determine the approximate scope of word problems in such computation lessons. The data below represent the approximate number of "whole" lessons that could be formed were the word problems on computation pages "accumulated" into whole lessons:

	Approximate Number of Lessons	
Series	**Grade 2**	**Grade 3**
Scott Foresman	4-6	4-6
Addison Wesley	---	---
DC Heath	1-4	2-4
Silver Burdett	---	---
Macmillan	1-4	2-6

The average number of lessons focused on word problems involving multiplication/division in grade 4 remains about the same as that in grade 3. There are some changes, however, as problems that require two or more kinds of computation (e.g., multiplication and then addition) come into focus. There is also more of a "consumer" orientation involving tax, unit price, etc.

Table 5

Grade 3: Fraction Recognition

Grade Level	Average Number of Lessons	Range	Average % of Instruction	Range	Most Common Location in Text
2	3.4	2-5	2.2%	1-4%	4 or 6
3	5.3	3-7.5	3%	2-4%	5 or 6
4	6.4	5-8	3.8%	3-5%	5 or 6

The most conspicuous aspects of the instruction on this skill are its amount--less than 4% of the instruction at any grade level is focused on recognition of fractions--and its location--almost always found in the final portions of the book. One textbook series does vary from the others by locating work on fractions near the middle of each grade level. (This series is also the only one that does not introduce decimals until the final sixth of the textbook at grade 4.)

Data from grade 2 are included in Table 5 just to fill out the picture of instruction across grade levels. The grade 2 instruction typically involves the recognition of halves, thirds (2/3, 1/3) and fourths (3/4, 2/4, 1/4) of regions.

All textbooks include regions first and then sets as models for fraction recognition. Three textbook series include some type of linear model (inch ruler, centimeter ruler, or number line), but one of these waits until grade 4 to do so. The slight increase in instruction in

grade 4 is tied to the introduction of mixed numbers (only one textbook includes mixed numbers in grade 3) and the extension of decimal recognition to hundredths.

Table 6

Grade 4: Multiplication and Division with 1-Digit Multipliers or Divisors

Grade Level	Average Number of Lessons	Range	Average % of Instruction	Range	Most Common Location in Text
3	7.6	5-10	4.2%	3-6%	5 or 6 (x) 5 or 6 (÷)
4	11.4	9.5-19	7.6%	5-10%	3 (x) 4 (÷)
5	6.6	5-8	3.6%	3-4%	2 (x) 2 or 3 (÷)

Data from grade 3 are included here to fill out the picture of how this skill develops. All textbooks at grade 3 locate instruction on multiplication and division by 1-digit numbers in the last two sixths of the textbook, usually right after they provide some work on hard multiplication or division facts.

Overall, the instruction that focuses on this skill moves from the end, to the middle, to the beginning sections of the textbook across grades 3, 4, and 5, respectively. Concurrently, the size of the multiplicands and the size of the quotient increases:

Size of Multiplicand	Number of Textbooks Using Multiplicands of Size Indicated		
	Grade 3	Grade 4	Grade 5
2 digit (33 X 4)	5		
3 digit (333 X 4)	1	5	
4 digit (3333 X 4)		3	5
5 digit (33,333 X 4)			1
6 digit (333,333 X 4)			2

Size of Quotient		Number of Textbooks Using Quotients of Size Indicated		
		Grade 3	Grade 4	Grade 5
2 digit	4)39 or 4)129	5		
3 digit	4)893 or 4)1293		5	5
4 digit	4)8932 or 4)12932		2	4
5 digit	4)89321 or 4)128,9321			1

For the most part, the textbook series have approximately the same number of lessons focusing on division as they do on multiplication in grades 3 and 4. In grade 3 there is a slight tendency for textbooks to have more multiplication lessons (an average of 4.2 multiplication lessons versus 3.6 division lessons). In grade 4 that relationship is reversed (an average of 6.2 multiplication lessons versus 7.6 division lessons).

By grade 5 there is a stronger tendency for textbooks to have more lessons on division with one-digit divisors than on multiplication with one-digit multipliers (an average of 2.2 multiplication lessons versus 4.4 division lessons). At this grade level the data are actually bimodal with two textbooks having four more division lessons than multiplication lessons and three textbooks having one more division lesson.

One aspect that seems clear from Table 6 is that the "brunt" of instruction on one-digit multipliers and divisors comes in grade 4. By grade 5, instruction moves well into 2 and 3-digit multipliers and divisors.

Table 7

Grade 4: Equivalent Fractions

Grade Level	Average Number of Lessons	Range	Average % of Instruction	Range	Most Common Location in Text
3*	1	1	.56%	.5-.6%	6
4	3.3	1.5-6	2%	1-4%	5 or 6
5	2.9	1.5-4	1.6%	1-2%	4

*Only three of the five series included any instruction in grade 3; two of those included instruction with models only, an extension of fraction recognition.

There really isn't much instruction focused directly on finding (or recognizing) equivalent fractions. Only one textbook allots more than 1% or 2% of its instruction time at any one grade. The one textbook that does spend more time on this skill, and that only at grade 4, overlaps the instruction with recognition of fractions for regions--in this case, providing two fractions rather than one. This same series is also the only one that includes instruction on equivalent fractions in the first half of the textbook. The dearth of instruction becomes more profound when one realizes that work on equivalent fractions includes finding fractions in higher terms, finding equivalent fractions with common denominators (essential for addition/subtraction and comparisons) and finding fractions in lowest terms (essential if one needs to simplify answers to computation).

In summary, adequate learning opportunities on pivotal skills is important if students are to be prepared to participate successfully in instruction in succeeding grade levels. The extent to which such opportunity is provided depends greatly on how instruction is organized in textbooks and how teachers use the organization. The adequacy varies from skill to skill. In terms of instructional "bulk," one of the pivotal skills has 10.6% of the textbook at the important grade level devoted to its development. Four of the skills have between 5% and 8% of the textbook devoted to them and two of the skills have less than 5%. In terms of instructional "location," instruction on two of these skills is found in the final third of the textbook, and may not be presented to

students at all. The instruction on two other skills is fragmented throughout the textbook, with no time allotted for concentrated practice.

The current structure of instruction in elementary mathematics includes elements of risk that many students will not get the instruction on pivotal skills that they need--when they need it. This does not mean that there is an optimal way to organize chapters in textbooks that current writers of textbooks have somehow missed. Some topics must be first, and others must come last in a sequence of lessons. But it is important to acknowledge that on some pivotal skills teachers must be provided additional instructional resources to "shore up" the instruction they are currently providing. The fact that an important topic will be reviewed next year does not provide enough insurance against instructional risk. Modest attention to the avoidance of instructional risk can effectively avoid much ineffective remedial instruction later.

References

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Freeman, D. J., et al. The influence of different styles of textbook use on instructional validity of standardized tests. Journal of Educational Measurement, 1983, 20(3), pp. 259-270.

APPENDIX A

This appendix contains a set of tables which describes the instruction focused on pivotal skills in each textbook series analyzed and at each appropriate grade level. The summary data in the tables in the body of this technical note was calculated from the "raw data" found in the following tables.

Table A Addition and Subtraction Algorithms with Regrouping
Grade 2 Textbooks

Table B Addition and Subtraction Algorithms with Regrouping
Grade 3 Textbooks

Table C Word Problems involving Addition/Subtraction
Grade 2 Textbooks

Table D Word Problems involving Addition/Subtraction
Grade 3 Textbooks

Table E Multiplication/Division Hard Facts
Grade 3 Textbooks

Table F Multiplication/Division Hard Facts
Grade 4 Textbooks

Table G Word Problems involving Multiplication/Division
Grade 3 Textbooks

Table H Word Problems involving Multiplication/Division
Grade 4 Textbooks

Table I Fraction Recognition
Grade 3 Textbooks

Table J Fraction Recognition
Grade 4 Textbooks

Table K Multiplication and Division Algorithms with 1-Digit Multipliers or Divisors
Grade 4 Textbooks

Table L Multiplication and Division Algorithms with 1-Digit Multipliers or Divisors
Grade 3 Textbooks

Table M Multiplication and Division Algorithms with 1-Digit Multipliers or Divisors
Grade 5 Textbooks

Table N Equivalent Fractions
Grade 4 Textbooks

Table O Equivalent Fractions
Grade 5 Textbooks

NOTE: The fourth column in each table shows the relative location of the relevant pivotal-skill instruction. This relative location is determined as follows:

1. The first decimal number represents the lesson number on which instruction begins, divided by the total number of lessons in the textbook.

2. The second decimal number represents the lesson number on which the instruction ends, divided by the total number of lessons in the textbook.

Other types of instruction are quite often interspersed within pivotal skill instruction; so, the numbers in the fourth column simply represent the part of the textbook within which pivotal skill instruction falls.

Table A

Grade 2
Addition and Subtraction Algorithms with Regrouping
Grade 2 Textbooks

Textbook Series	Number of Lessons	Percent of Instruction	Relative Location	Size of Numbers
Scott Foresman	26	16%	.59 - .92	2 and 3 digit
Addison Wesley	16	10%	.56 - .75	2 digit
D. C. Heath	15	9%	.45 - .54*	2 digit
Silver Burdett	10	7%	.9 - 1.0	2 digit
Macmillan	16	11%	.55 - .83	2 and 3 digit

*There is also "non-core" instruction which focuses on 3-digit numbers at the end of the text (.91-.96).

Table B

Grade 2
Addition and Subtraction Algorithms with Regrouping
Grade 3 Textbooks

Textbook Series	Number of Lessons	Percent of Instruction	Relative Location	Size of Numbers
Scott Foresman	19	11%	.25 - .64	2 or 3 digit
Addison Wesley	7	4%	.19 - .24	2 or 3 digit
D. C. Heath	13.5	8%	.19 - .34	2, 3 or 4 digit
Silver Burdett	26	15%	.25 - .51	2, 3 or 4 digit
Macmillan	17	10%	.21 - .37	2, 3 or 4 digit

Table C

Grade 2
Word Problems Involving Addition/Subtraction
Grade 2 Textbooks

Textbook Series	Number of Lessons*	Percent of Instruction	Relative Location**	Type of Problems***
Scott Foresman	9	6%	.23 - .91	r +, t
Addison Wesley	7.5	5%	.41 - .79	r, w +, t, c
D. C. Heath	10	7%	.11 - .96	r +, t, c
Silver Burdett	10.5	7%	.13 - .99	r, w +, t, c
Macmillan	6	4%	.5 - .28 .84	r +, t, c

*This column reflects the number of "two-page" lessons; thus if a program has a 1-page lesson (1/2 lesson), it is counted as a .5 lesson here, and two 1-page lessons are counted as 1 lesson.

**Word problem instruction is scattered intermittently throughout the textbook in most series.

***r = rebus problems
w = word problems
+ = addition problems
t = take away
c = comparison subtraction

Table D

Grade 2
Word Problems Involving Addition/Subtraction
Grade 3 Textbooks

Textbook Series	Number of Lessons*	Percent of Instruction	Relative Location*	Type of Problems**
Scott Foresman	10	6%	.11 - .99	w +, t, c
Addison Wesley	20	12%	.07 - .91	w +, t, c
D. C. Heath	15.5	10%	.04 - .98	r, w +, t, c
Silver Burdett	21	12%	.09 - .98	w +, t, c
Macmillan	9	5%	.04 - .56	w +, t, c

*Note: See Table C for descriptive information about columns 2 and 4.

**w = word problems
+ = addition problems
t = take away
c = comparison subtraction

Table E

Grade 3
Multiplication/Division Hard Facts
Grade 3 Textbooks

Textbook Series	Number of Lessons	Percent of Lessons	Relative Location	Hard Fact Focus*
Scott Foresman	13	7%	.47-.50(x) .73-.76(÷) .77-.78(x/÷)	1/3
Addison Wesley	11	7%	.40-.43(x) .80-.82(÷)	1/2 to 1/3
D. C. Heath	8	5%	.76-.81(x/÷)	1/2 to 1/3
Silver Burdett	5	3%	.64-.65(x) .72-.73(÷)	1/2
Macmillan	11	6%	.54-.56(x) .65-.66(÷)	1/2 to 1/3

*The most common form of instruction for hard facts is to do x6; then x7; then x8; then x9 or ÷6; then ÷7; then ÷8; then ÷9. Thus, a sample lesson would include 6x2, 6x3, 6x4, 6x5, 6x6, 6x7, 6x8, 6x9. The fractions in the last column represent the approximate portion of each lesson which focuses on the hard facts in which both factors are greater than 5 (i.e.. how many of the problems in a lesson are the hard facts).

Table F

Grade 3
Multiplication/Division Hard Facts
Grade 4 Textbooks

Textbook Series	Number of Lessons	Percent of Lessons	Relative Location	Hard Fact Focus*
Scott Foresman	6	3%	.38(x) .60(÷) .61(x)	1/3
Addison Wesley	5	3%	.22-.24(x) .28(÷)	1/2 to 1/3
D. C. Heath	4	2%	.31-.32(x) .37-.38(÷)	1/2
Silver Burdett	3.5	2%	.28-.30(x/÷)	1/2 to 1/3
Macmillan	5	3%	.25-.26(x) .30(÷)	1/4 to 1/5

*See Table E for a description of the data in this column.

Table G

Grade 3
Word Problems Involving Multiplication/Division
Grade 3 Textbooks

Textbook Series	Number of Lessons	Percent of Lessons	Relative Location*	Number of Problems X	P÷	M÷**
Scott Foresman	7.5	4%	.51-.99	30	14	35
Addison Wesley	11	7%	.44-.90	40	8	4
D. C. Heath	11.5	7%	.45-.95	40	8	32
Silver Burdett	9	5%	.58-.98	41	9	20
Macmillan	6	4%	.52-.91	22	14	7

*Lessons are scattered intermittently throughout the last half of the textbook in most series.

**The data in this column represent the approximate number of word problems, of each type described below, found in half (one page) or whole (two page) lessons in each textbook.

x = multiplication word problems

P÷ = partitive division word problems (54 children; 6 teams; how many on each team)

M÷ = measurement division word problems (54 children; 9 on each team; how many teams)

Table H

Grade 3
Word Problems Involving Multiplication/Division
Grade 4 Textbooks

Textbook Series	Number of Lessons	Percent of Lessons	Relative Location*	Number of Problems X	P+	M+	O+**
Scott Foresman	9.5	5%	.36-.98	42	2	15	16(A)
Addison Wesley	10	6%	.30, .50, .62	27	4	10	
D. C. Heath	5.5	3%	.33-.90	24	5	2	1(A)
Silver Burdett	13.5	8%	.27-.98	99	20	28	2(A) 2(R)
Macmillan	7	4%	.27-.96	30	24	6	

*Lesson are scattered intermittently throughout textbook in most series.

**X = Multiplication word problems
P+ = partitive division word problems
M+ = measurement division word problems
O+ = other division;
A = "Average" word problems
R = "Rate" word problems

Table 1

Grade 3
Fraction Recognition
Grade 3 Textbooks

Textbook Series	Number of Lessons	Percent of Lessons	Relative Location	Type of Instruction*
Scott Foresman	5	3%	.80-.83	f, d r, s
Addison Wesley	7	4%	.90-.94	f, d r, s, n
D. C. Heath	3	2%	.58-.60	f r, s
Silver Burdett	7.5	4%	.77-.82	f, d r, s, n
Macmillan	4	2%	.92-.97	f, d r, s

*f = fractions
d = decimals
r = region models
s = set models
n = number line models (may include rulers)

Table J

Grade 3
Fraction Recognition
Grade 4 Textbooks

Textbook Series	Number of Lessons	Percent of Lessons	Relative Location*	Type of Instruction**
Scott Foresman	8	4%	d .23 f .80-.83	f, d r, s, n
Addison Wesley	8	5%	f .69-.72 d .82-.83	f, d r, s, n
D. C. Heath	6	4%	f .41-.43 d .92-.94	f, d r, s
Silver Burdett	5	3%	f .76-.78 d .79	f, d r, s
Macmillan	5	3%	f .61-.63 d .93	f, d r, s

*f = fractions
d = decimals
**f = fractions
d = decimals
r = region models
s = set models
n = number line model (may include rulers)

Table K

Grade 4
Multiplication and Division Algorithms
with 1-Digit Multipliers/Divisors
Grade 4 Textbooks

Textbook Series	Number of Lessons	Percent of Lessons	Relative Location	Size of Numbers*
Scott Foresman	12.5	7%	.39-.43(x) .63-.65(÷)	2 or 3 digit
Addison Wesley	9.5	5%	.38-.42(x) .54-.57(÷)	2 or 3 digit
D. C. Heath	12	8%	.64-.68(x) .71-.75(÷)	2, 3 or 4 digit
Silver Burdett	14	8%	.32-.37(x) .46-.50(÷)	2, 3 or 4 digit
Macmillan	19	10%	.31-.32(÷) .45-.48(x) .53-.57(÷)	2, 3 or 4 digit

*For multiplication these numbers represent the size of the multiplicand. For division these numbers represent the size of the quotient.

Table L

Grade 4
Multiplication/Division Algorithms
with 1-Digit Multipliers/Divisors
Grade 3 Textbooks

Textbook Series	Number of Lessons	Percent of Lessons	Relative Location	Size of Numbers*
Scott Foresman	9.5	5%	.87-.90(x)	2 digit
			.97-.98(÷)	2 digit
Addison Wesley	10	6%	.66-.69(x)	2 or 3 digit
			.82-.85(÷)	2 digit
D. C. Heath	5	3%	.94-.95(x)	2 digit
			.96-.97(÷)	2 digit
Silver Burdett	6.5	3%	.67-.68(x)	2 digit
			.75-.77(÷)	2 digit
Macmillan	7	4%	.83-.84(x)	2 digit
			.87-.88(÷)	2 digit

*For multiplication these numbers represent the size of the multiplicand. For division these numbers represent the size of the quotient.

Table M

Grade 4
Multiplication/Division Algorithms
with 1-Digit Multipliers/Divisors
Grade 5 Textbooks

Textbook Series	Number of Lessons	Percent of Lessons	Relative Location	Size of Numbers*
Scott Foresman	6	3%	.11(x) .35-.38(÷)	2, 3 or 4 digit 2 or 3 digit
Addison Wesley	5	3%	.39(x) .50-.51(÷)	2, 3 or 4 digit 2, 3 or 4 digit
D. C. Heath	7	4%	.19-.20(x) .27-.30(÷)	2 to 6 digit 2, 3 or 4 digit
Silver Burdett	7	4%	.18-.19(x) .25-.27(÷)	2 to 6 digit 2, 3 or 4 digit
Macmillan	8	4%	.22(x) .30-.35(÷)	2 to 5 digit 2 to 5 digit

*For multiplication these numbers represent the size of the multiplicand. For division these numbers represent the size of the quotient.

Table N

Grade 4
Equivalent Fractions
Grade 4 Textbooks

Textbook Series	Number of Lessons	Percent of Lessons	Relative Location
Scott Foresman	2	1%	.88
Addison Wesley	3	2%	.71-.72
D. C. Heath	6	4%	.43-.48
Silver Burdett	1.5	1%	.79, .87
Macmillan	4	2%	.63-.67

Table O

Grade 4
Equivalent Fractions
Grade 5 Textbooks

Textbook Series	Number of Lessons	Percent of Lessons	Relative Location
Scott Foresman	4	2%	.57-.58
Addison Wesley	4	2%	.68-.69
D. C. Heath	3	2%	.48-.49
Silver Burdett	1.5	1%	.52
Macmillan	2	1%	.59

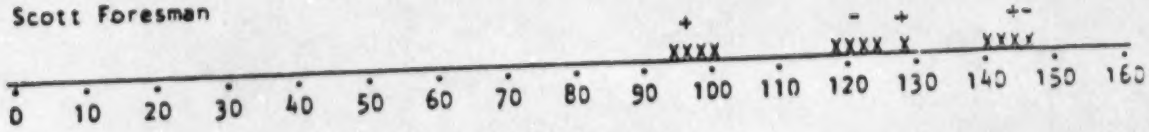
APPENDIX B

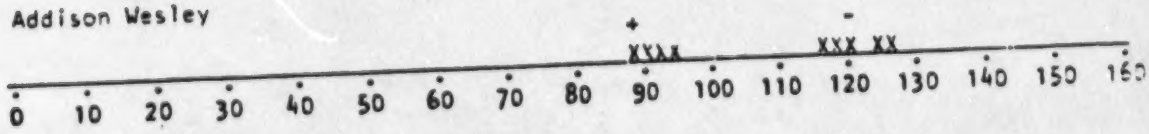
This appendix contains a set of figures which show "usually" where the pivotal skill instruction is found in each textbook series analyzed. Each figure contains five time lines (one for each of the textbook series). Each time line (see example below) shows:

1. The approximate number of lessons (2-page units) in the textbook at that grade level (approximately 160 in the example); and
2. the location of pivotal skill instruction, with each X representing 1 or 2 lessons (the first addition instruction is found in lessons 94-101 in the example).

Example

ADDITION and SUBTRACTION ALGORITHMS WITH REGROUPING
GRADE 2 Textbooks





The following figures are found in this appendix.

Figure 1 Addition and Subtraction Algorithms with Regrouping Grade 2 Textbooks

Figure 2 Addition and Subtraction Algorithms with Regrouping Grade 3 Textbooks

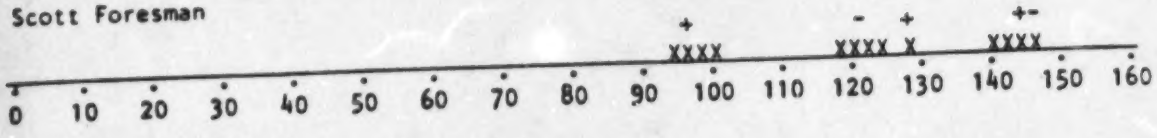
Figure 3 Basic Word Problems Involving Addition and Subtraction Grade 2 Textbooks

Figure 4 Basic Word Problems Involving Addition and Subtraction Grade 3 Textbooks

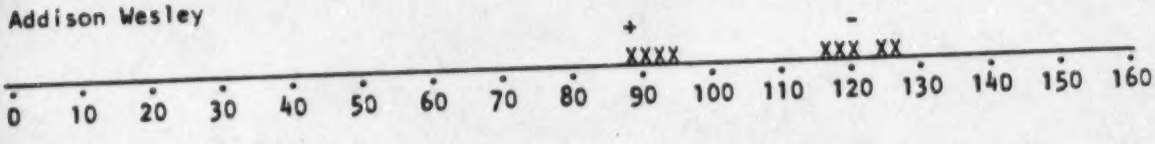
FIGURE 1

ADDITION and SUBTRACTION ALGORITHMS WITH REGROUPING
GRADE 2 Textbooks

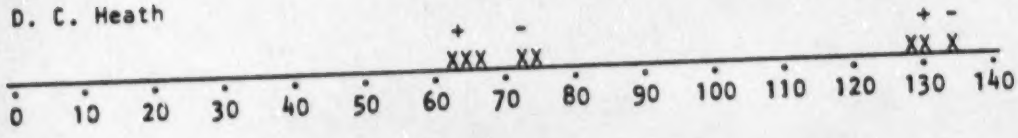
Scott Foresman



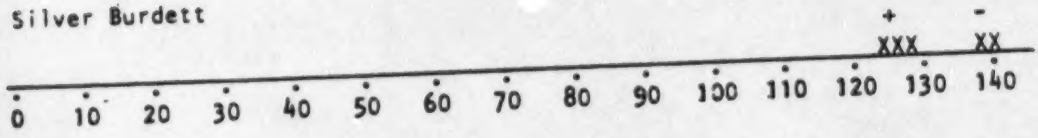
Addison Wesley



D. C. Heath



Silver Burdett



MacMillan

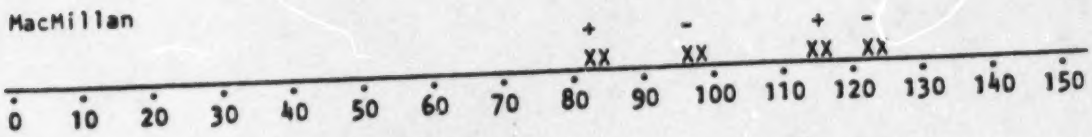
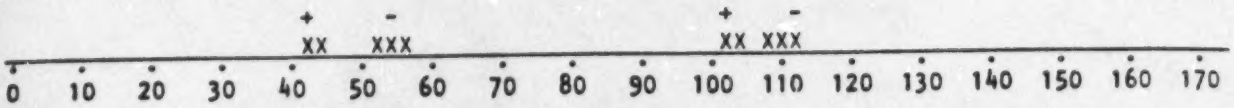


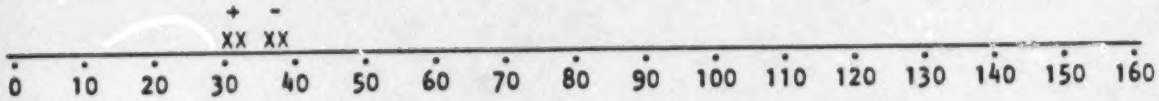
FIGURE 2

ADDITION AND SUBTRACTION ALGORITHMS WITH REGROUPING
GRADE 3 Textbooks

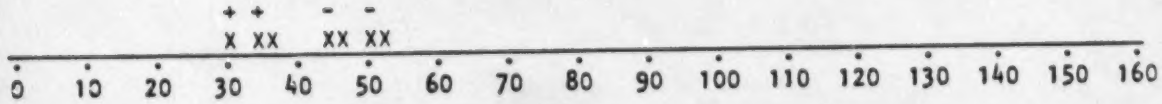
Scott Foresman



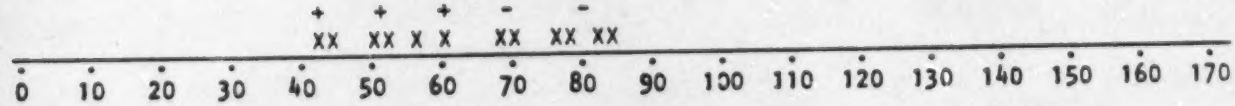
Addison Wesley



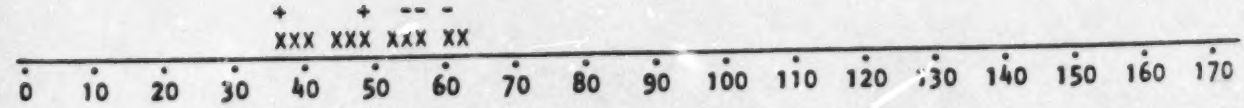
D. C. Heath



Silver Burdett



MacMillan

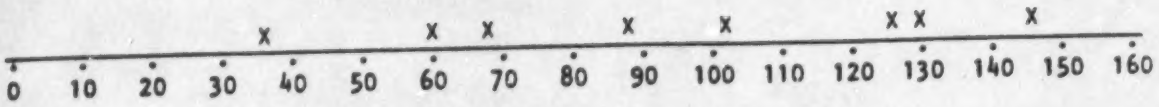


Note: Three of the series (Addison Wesley, Silver Burdett, and MacMillan) treat addition and subtraction separately. The other two series contain one or two lessons, at the end of instruction on subtraction, that provide mixed practice on addition and subtraction.

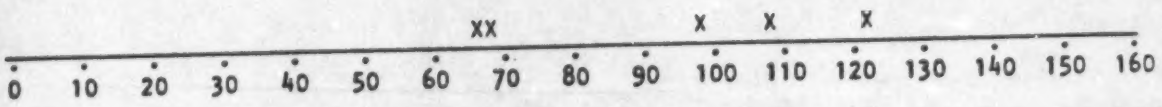
FIGURE 3

BASIC WORD PROBLEMS INVOLVING ADDITION/SUBTRACTION
GRADE 2 Textbooks

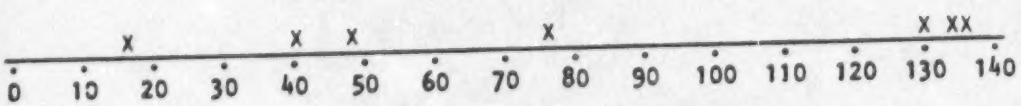
Scott Foresman



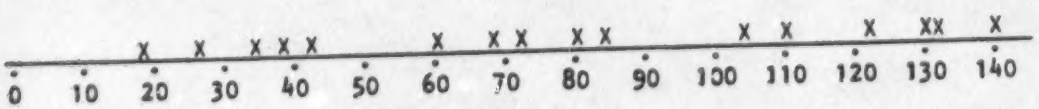
Addison Wesley



D. C. Heath



Silver Burdett



MacMillan

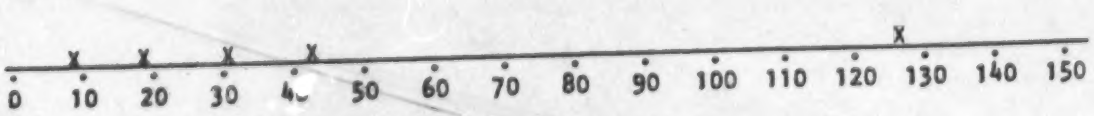
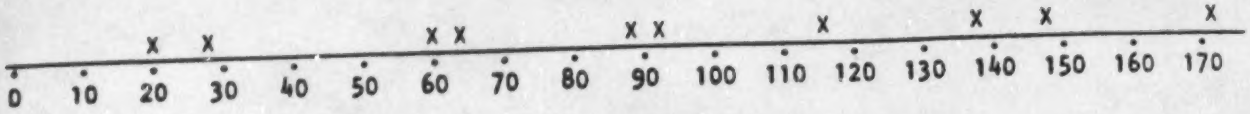


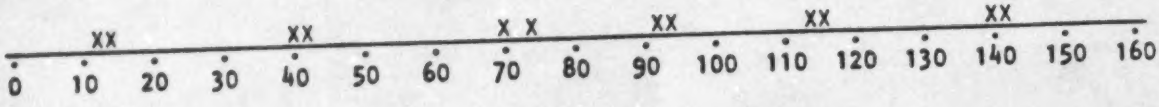
FIGURE 4

BASIC WORD PROBLEMS INVOLVING ADDITION/SUBTRACTION
GRADE 3 Textbooks

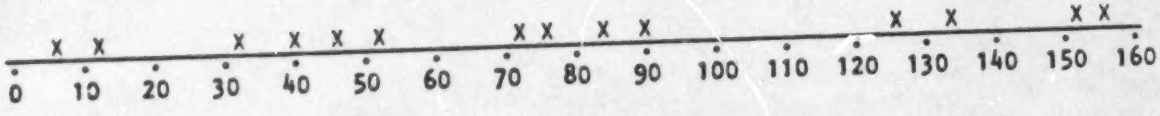
Scott Foresman



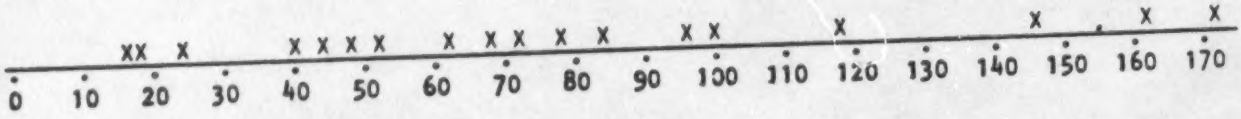
Addison Wesley



D. C. Heath



Silver Burdett



MacMillan

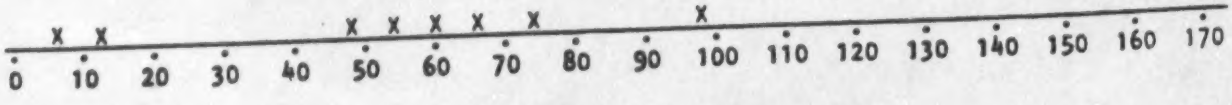


FIGURE 5

MULTIPLICATION/DIVISION "HARD" FACTS
GRADE 3 Textbooks

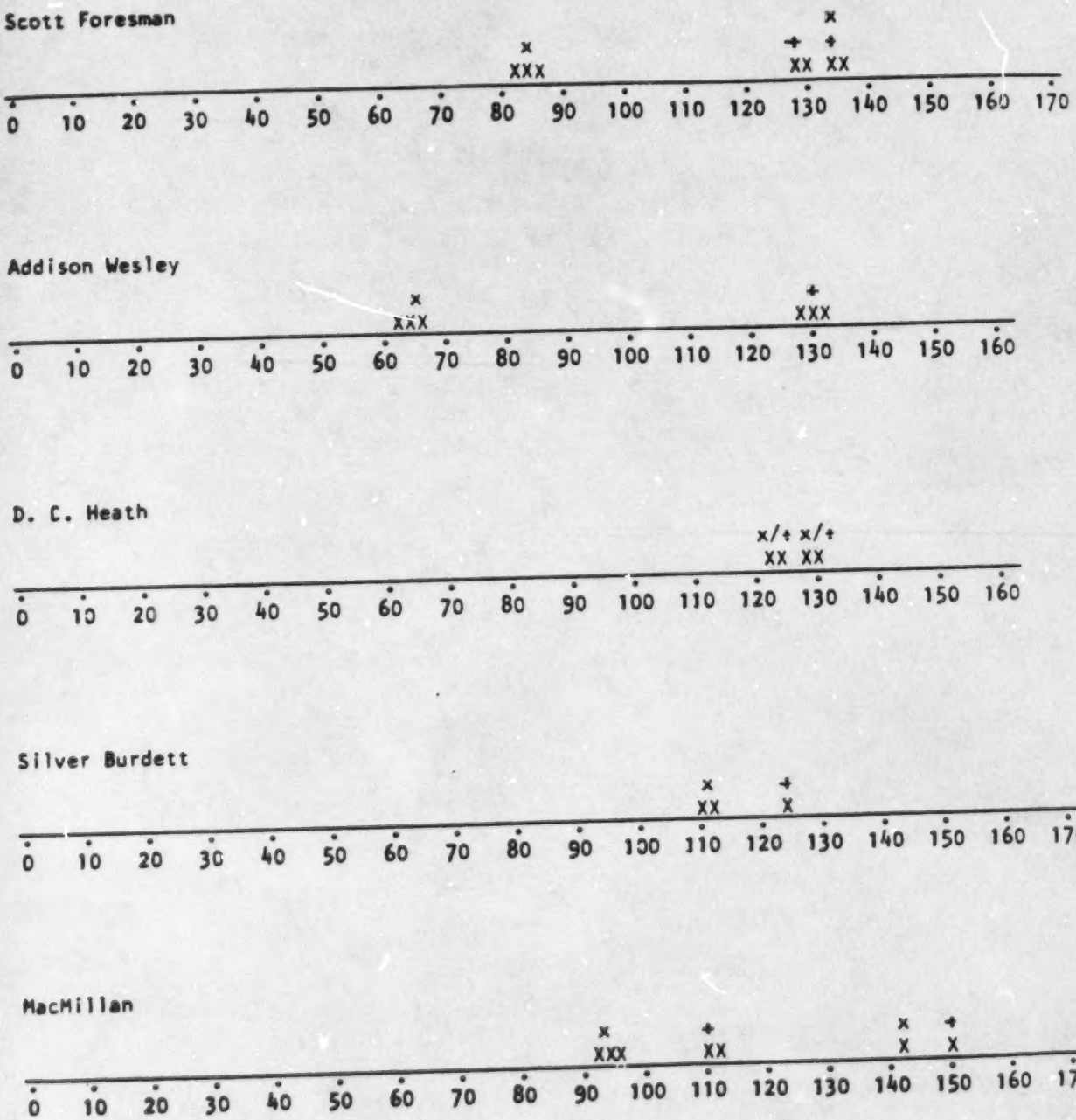


FIGURE 6

MULTIPLICATION/DIVISION "HARD" FACTS
GRADE 4 Textbooks

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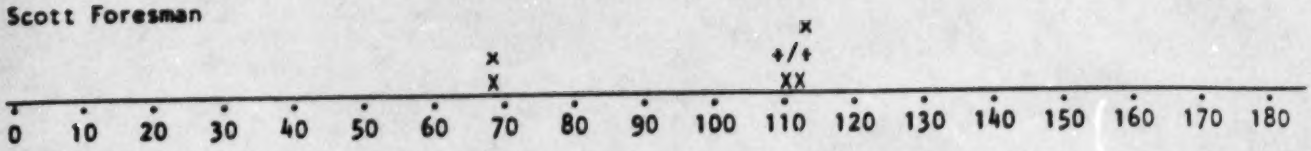
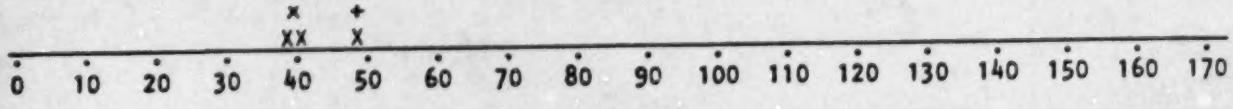
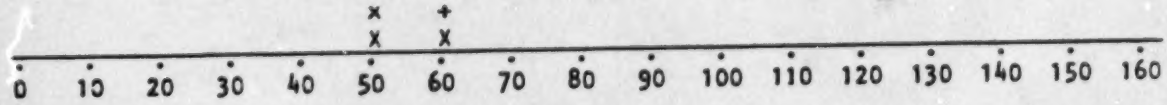


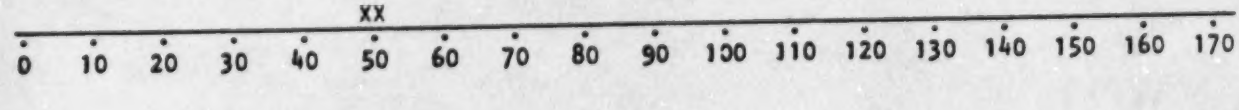
Addison Wesley



D. C. Heath



Silver Burdett



MacMillan

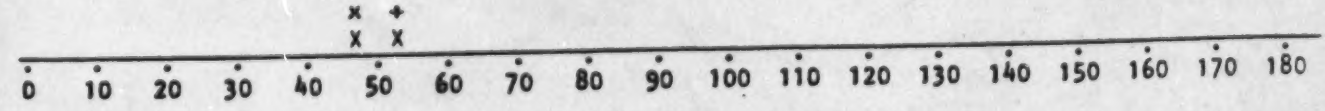
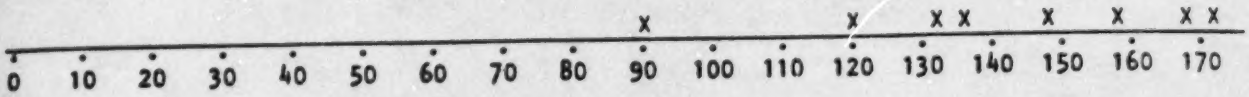


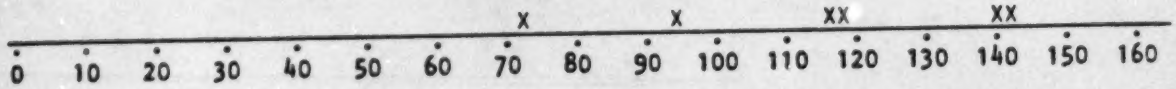
FIGURE 7

BASIC WORD PROBLEMS INVOLVING MULTIPLICATION/DIVISION
GRADE 3 Textbooks

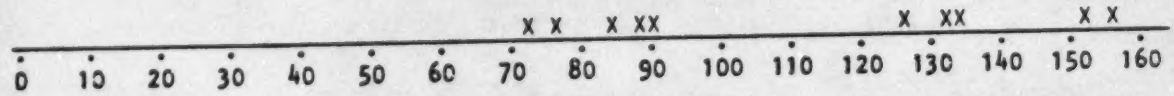
Scott Foresman



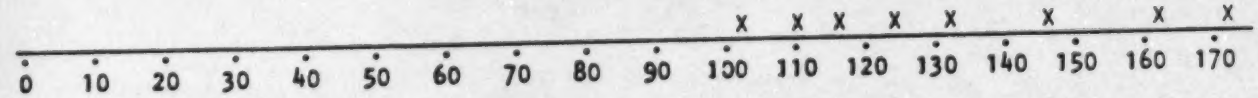
Addison Wesley



D. C. Heath



Silver Burdett



MacMillan

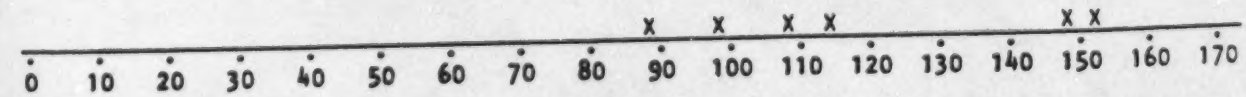
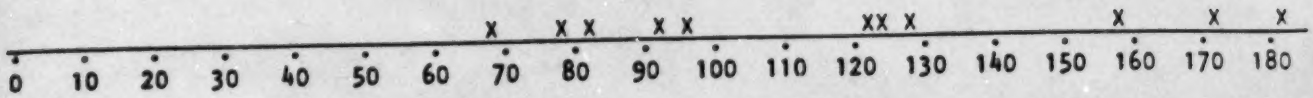


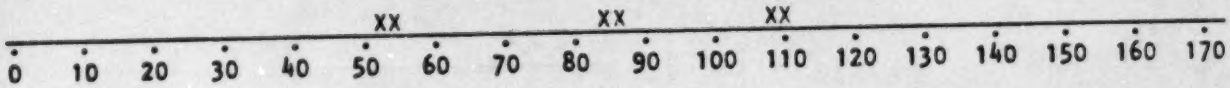
FIGURE 8

BASIC WORD PROBLEMS INVOLVING MULTIPLICATION/DIVISION
GRADE 4 Textbooks

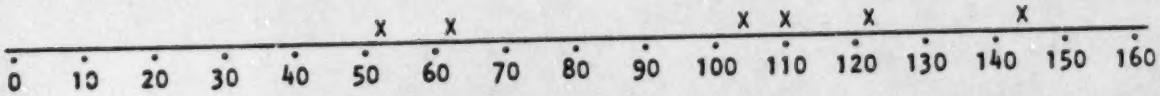
Scott Foresman



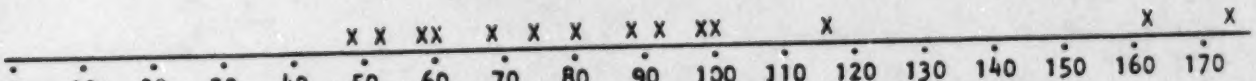
Addison Wesley



D. C. Heath



Silver Burdett



MacMillan

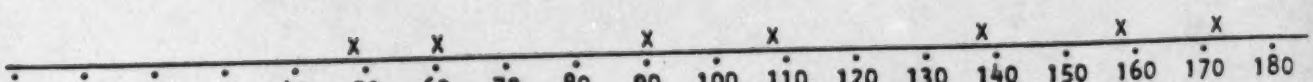
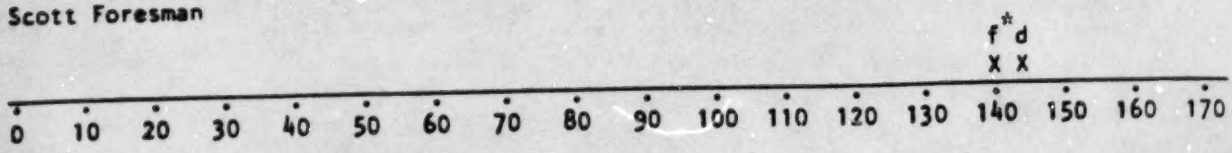


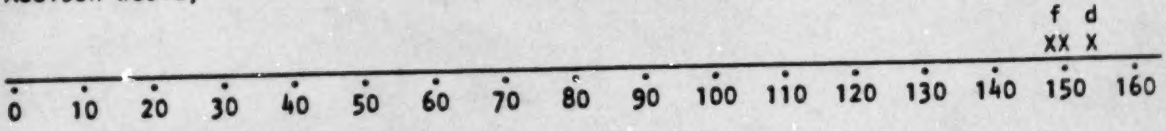
FIGURE 9

FRACTION RECOGNITION
GRADE 3 Textbooks

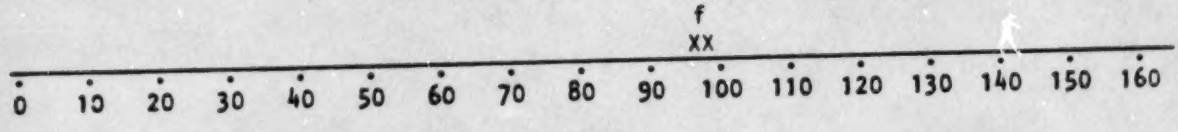
Scott Foresman



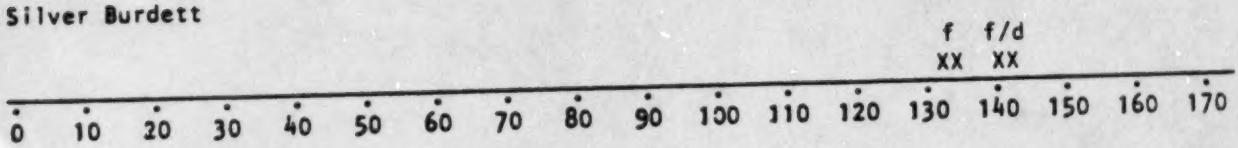
Addison Wesley



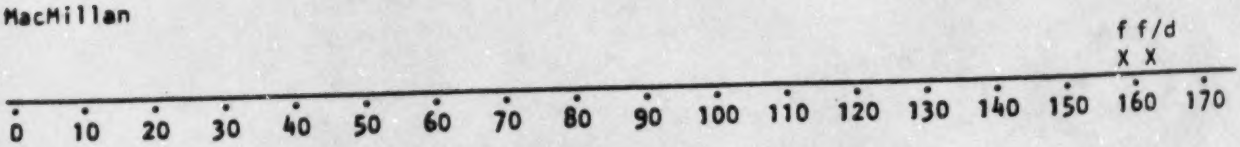
D. C. Heath



Silver Burdett



MacMillan

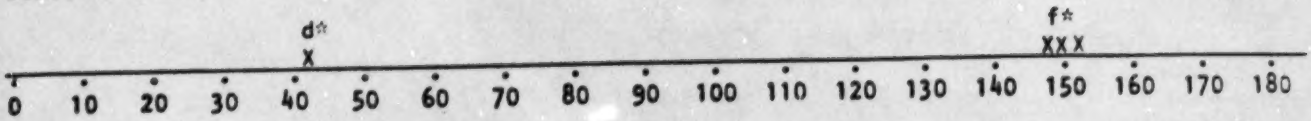


*f = common fractions
d = decimals

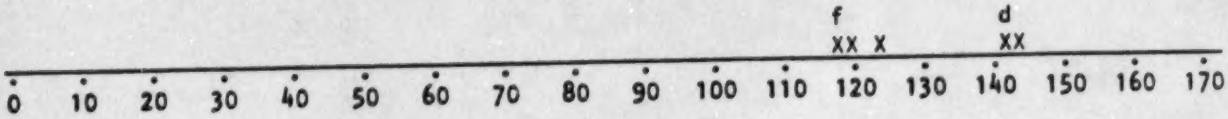
FIGURE 10

FRACTION RECOGNITION
GRADE 4 Textbooks

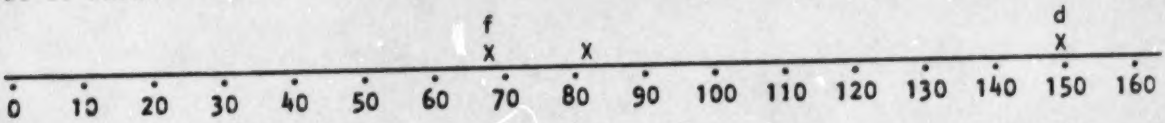
Scott Foresman



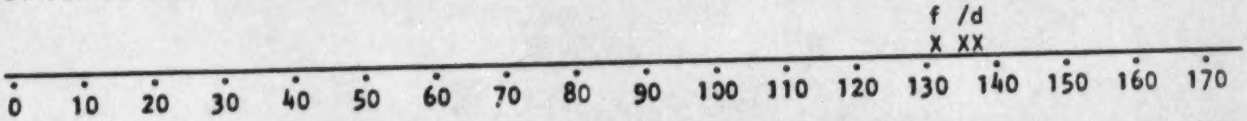
Addison Wesley



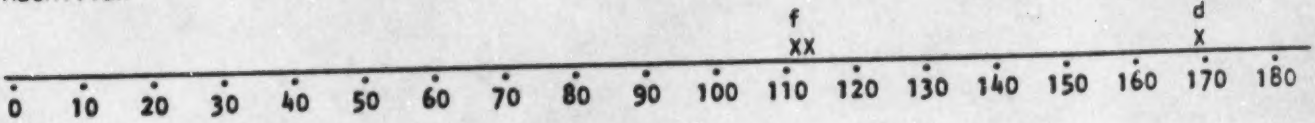
D. C. Heath



Silver Burdett



MacMillan

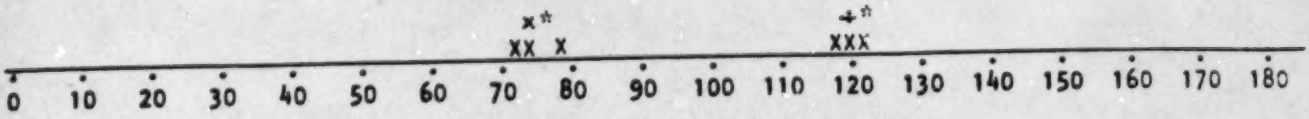


*d = decimals
f = common fractions

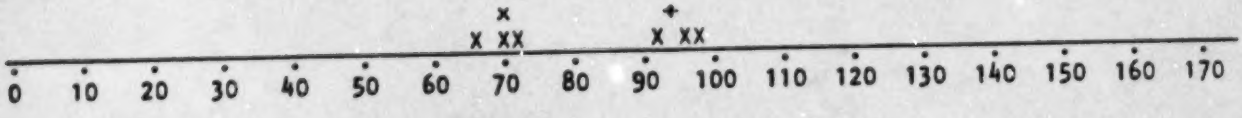
FIGURE 11

MULTIPLICATION/DIVISION ALGORITHMS WITH 1-DIGIT MULTIPLIERS/DIVISORS
GRADE 4 Textbooks

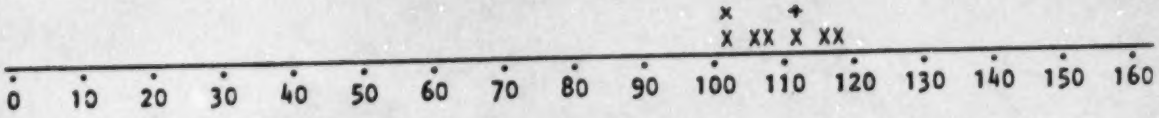
Scott Foresman



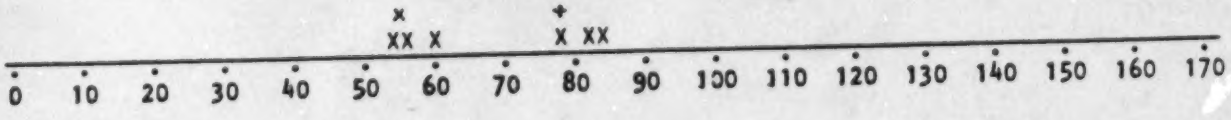
Addison Wesley



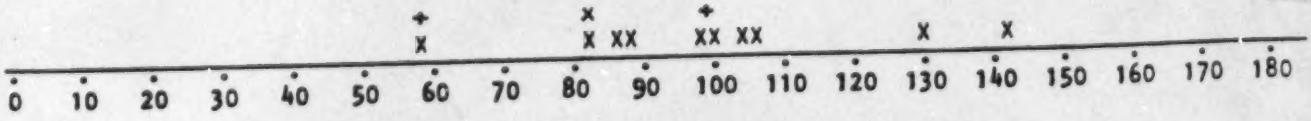
D. C. Heath



Silver Burdett



MacMillan

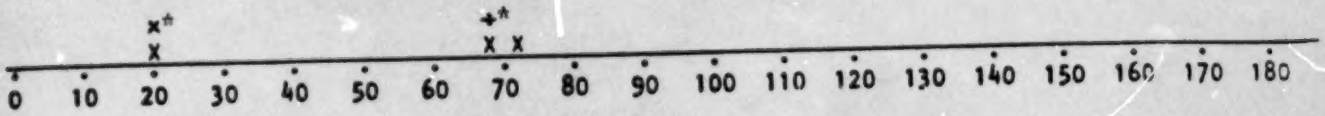


* x = multiplication
  + = division

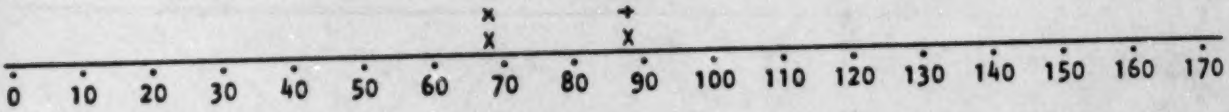
FIGURE 12

MULTIPLICATION/DIVISION ALGORITHMS WITH 1-DIGIT MULTIPLIERS/DIVISORS
GRADE 5 Textbooks

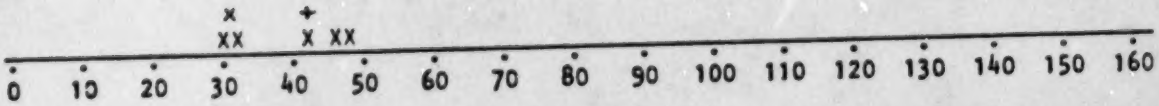
Scott Foresman



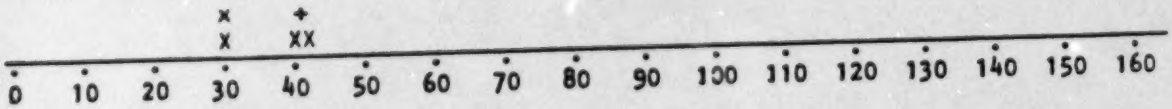
Addison Wesley



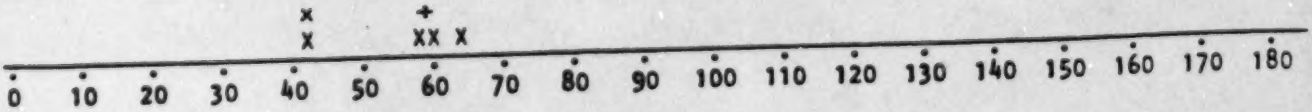
D. C. Heath



Silver Burdett



MacMillan

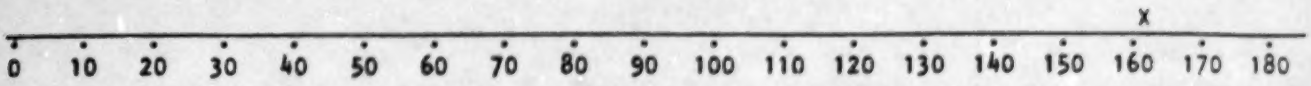


*x = multiplication
÷ = division

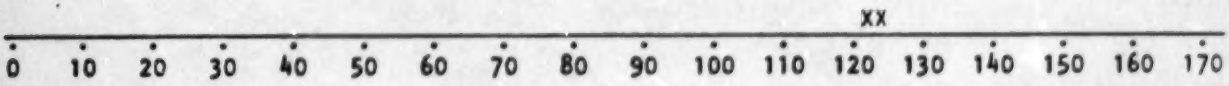
FIGURE 13

EQUIVALENT FRACTIONS
GRADE 4 Textbooks

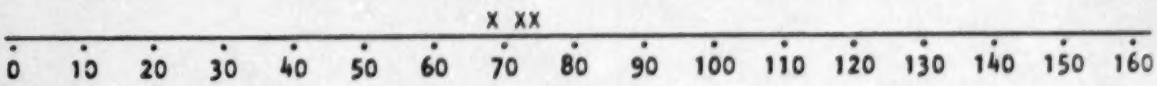
Scott Foresman



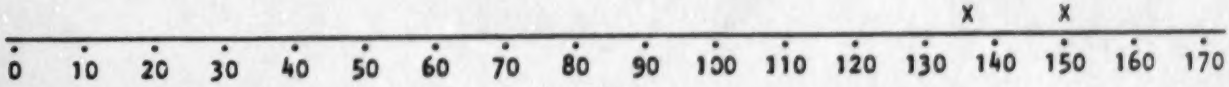
Addison Wesley



D. C. Heath



Silver Burdett



MacMillan

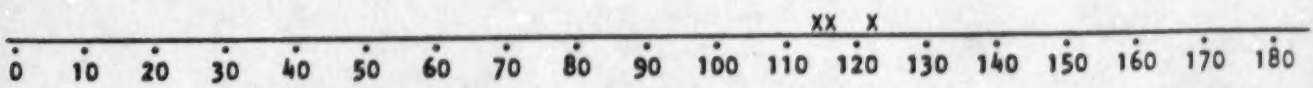
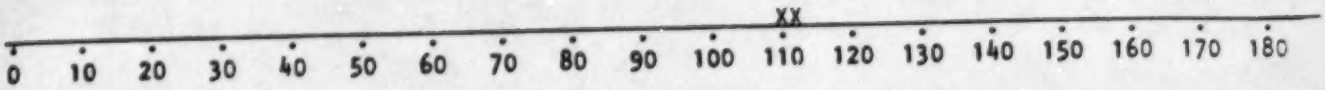


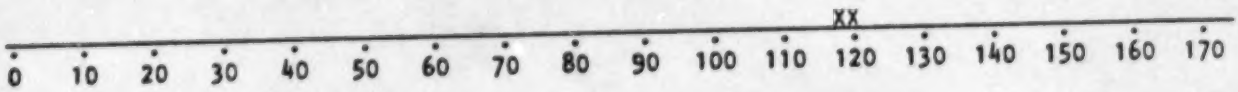
FIGURE 14

EQUIVALENT FRACTIONS
GRADE 5 Textbooks

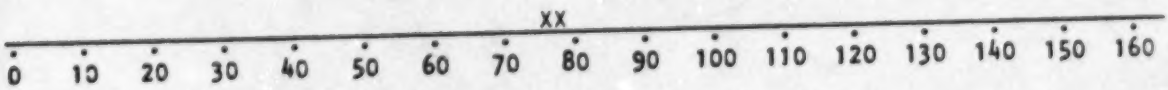
Scott Foresman



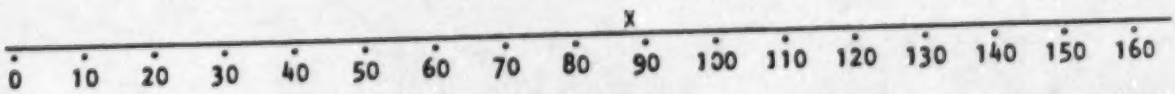
Addison Wesley



D. C. Heath



Silver Burdett



MacMillan